Mathematics for Psychoanalysis.

Brouwer's Intuitionism from Descartes to Lacan

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No math, no science

It is still astonishing how much of the formalism of quantum mechanics was "already written" in the theory of the functions of complex variables a long time before the experimental data was known. Not to mention how much of Einstein's theory of relativity was pre-written in the tensor calculus of Ricci Curbastro and Levi Civita, or how much of Darwin's theory of speciation is written into the theory of chaos. It doesn't seem to be going to far out on a limb to state that mathematics is the transcendental condition of science. In other words, mathematics is the necessary condition that makes science possible. To make a slogan out of it, we might say: *no math, no science*. On this point, perhaps because he was caught up in his own cognitive fantasies, Kant was misled: mathematics is not *a priori* synthesis. It is rather the *a priori* that allows the synthesis of diverse experimental and theoretical data, because it is already predisposed to fulfil this function. Mathematics is knowledge, perhaps not yet known, as Heidegger says in *The Age of the World Picture* [1]. It is knowledge, not necessarily available in a conceptual form, that permits knowing more.

Here I would like to prove the reasonableness of this thesis by presenting how much of the logic of the unconscious has already been written, certainly unwittingly, in Brouwer's intuitionistic logic.

My approach is justified *a posteriori* by the results that it produces. Thus, without justifying myself beforehand, I assume that the intuitionist logic is an epistemic logic.

Traditionally the axiomatic systems of epistemic logic are constructed by addition: to the axioms of classical, propositional, or predicative logic, are added those extra that implicitly define the properties of the epistemic operator: "I know that *X*" or "I am aware that *X*", where *X* is a propositional variable. It is therefore at first sight surprising to think of as epistemic a logic that subtracts axioms from classical logic rather than adding them to it. In fact, intuitionism, suspends two fundamental axioms of classical logic: the axiom of the excluded third (*A vel not A*) and the equivalence between existential and universal operators by means of a double negation: "there does not exist one that does not" is not equivalent to "all".

The first justification of intuitionism as an epistemic logic is its wealth of operators. In intuitionist logic, the three binary logical operators AND, OR, IF do not express themselves through the others by means of the negative (NOT) as in classical logic, practically becoming one only, but rather each of them maintains its own individuality that distinguishes it from the others. Further, the two quantifiers, universal and existential, are truly two and not just the same operator in different versions.

But let's look at the details. The driving idea of my work is that a classical non-intuitionist thesis, like the excluded third, becomes an epistemic operator. I will begin precisely from the …

The Excluded Third

The excluded third is written *X vel not X*. It is the simplest classical non-intuitionist thesis. What does it mean to say that it becomes an operator? It means that and endomorphism ε is defined and that every statement *X* is transformed in *X vel not X*, and written ε*X*. What are the properties of ε? There are theorems (marked by Frege's symbol for judgment |—) and non-theorems (marked by Frege's symbol reversed —|), which we will now analyse in detail, and which characterise ε as an epistemic operator. In particular, such theorems show that ε shares many properties of a particular kind of knowledge: unconscious knowledge.

Kolmogorov's lemma

If X does not contain universal quantifiers, then $|\!\!—$ *non non εX.*

This is an important lemma, in as much as it demonstrates a kind of weak equivalence between classical logic and intuitionist logic. In fact, the declarative *X*, without its universal quantifiers, is a classic thesis if and only if its double negation is an intuitionistic thesis. In this sense, the lemma takes the coherence of classical logic back to that of intuitionistic logic, providing that the double negation is understood as "it cannot not be true that".

The epistemic interpretation is no less interesting: "it is not possible to not know that". With this the intention is not to introduce omniscience, but to state that even if you don't know now, sooner or later, given adequate analytical work, you will come to know.

Two important factors are introduced by the lemma:

- The time of knowing. We don't know all and immediately. We come to know by means of a process of elaboration that takes time.
- The epistemic value of uncertainty. Knowing is generated from not knowing. The movement is typical of Descartes's *cogito*, which acquires the certainty of the existence of the subject beginning from systematic doubt, that is, from generalised uncertainty.

For the analyst, the lemma puts in focus an essential feature of unconscious knowledge that, before being knowledge is a knowing that is beyond the reach of consciousness, a knowledge that consciousness does not yet possess. It is knowledge that resides in the subject, where it produces effects: dreams, slips, neurotic symptoms, of which, however, the subject will become aware only when things are done, not before.

Socrates' theorem

Since it was dominated by Aristotelian logic, which was more interested in the transmission of the truth by means of a chain of deductions than in highlighting the knowledge embodied in it, antiquity has passed down to us very few epistemic theorems. There is practically a single one, that of Socrates, who states that he knows one thing only, which is, that he doesn't know anything. I will transcribe Socrates' theorem as a variant of Kolmogorov's lemma, where the epistemic operator ε takes the place of the first negative:

$$|\!\!— \varepsilon \text{ non } \varepsilon X.$$

For the analyst, the importance of this theorem lies in its moral connotations. The subject of the unconscious is never completely ignorant; it always knows something, for instance, it knows that it doesn't know. Faced with a completed action, the subject cannot justify himself by saying, "But I didn't know". "No", the psychoanalyst tells you, "you are always responsible, at least partially, for your actions, because you already knew something about what you did before you did it". It goes without saying that without this epistemic responsibility no psychoanalytical elaboration can begin. Psychoanalysis, before being a therapy, is an ethic: it takes care of the ethic that the subject had momentarily lost or weakened and can now, with analytical work, rediscover or reinforce.

Descartes's theorem

The entire movement of Cartesian doubt took place within an epistemic logic: that logic that begins with a variant of the principle of the excluded third, to be precise, the epistemic variant. In fact, Cartesian doubt can be expressed as an epistemic alternative: *I know or I don't know*, thus I am uncertain about my knowledge. From this uncertainty derives the certainty of the existence of the new subject: the modern subject of science. (I will come back to this argument below).

It is not surprising therefore that within intuitionistic logic, which transforms the excluded third into an epistemic operator, there is a place for a theorem that can justly be attributed to Descartes: *If I don't know, then I know*. In a formula:

$$|\!\!- \textit{non } \varepsilon X \textit{ seq } \varepsilon X.$$

The merit of this formulation is that it clarifies the nature of the knowledge at stake. We are not dealing with a bookish knowledge, written down in some manual. Descartes's theorem doesn't serve to pass some exam at the University. In effect, we are dealing with a subjective knowledge, one that the subject actually does not know, but which he will come to know with the work of the analysis. Descartes's theorem guarantees the passage from unconscious to conscious, without saying, fortunately or unfortunately, precisely how that passage effectively takes place: the pleasure of discovering that is left to the individual subject within his own analysis.

In passing I remark that routine clinical observation will affirm that the neurotic subject, especially of the obsessive type, will state beforehand the declarative "I don't know", when he is about to declare some truth about his own story. The analyst therefore deduces: "so he knows".

There is a second formulation – philonian – of Descartes's theory, no less interesting, in that it shows the affinity with Kolmogorov's lemma:

$$|\!\!- \textit{non non } \varepsilon X \textit{ vel } \varepsilon X.$$

If there were no time of knowledge, there would be a tautology: either you know or it isn't true that you don't know. But since there exists a time for learning, the theorem suggests a more complex epistemic event: either you know or it is not true that you will not know, sooner or later.

Idempotence or the conscience that doesn't add anything

The theorem of epistemic idempotence states that knowledge of knowledge is equivalent to knowledge. In a formula:

$$|\!\!- \varepsilon\varepsilon X \textit{ aeq } \varepsilon X.$$

If, following a suggestion by Odifreddi, I interpret the duplication of the operator as its "conscience" (see [2]), I obtain the decadence of the first Freudian topic. In effect, the distinction conscious/unconscious ceases to be operative, because the same knowledge is at work on both the conscious and the unconscious levels. The passage from one to the other is certain. It is only a question of time, even if when it will take place cannot be predicted.

Against ontology, or in favour of laity

Scholars discuss whether psychoanalysis is a science or not. Epistemic logic makes its contribution to the discussion by indicating what psychoanalysis is not: it is not religion, if it is intuitionism. In fact, in intuitionist logic the ontological argument is not valid. This means that from the existence of the object of which you know a property it is not possible to deduce that you know the existence of an object with that property: knowing the qualities of an object, as essential as they may be, is not sufficient to determine the knowledge of the existence of the object. In short, essence does not implicate existence:

$$-\!\!| \ (\exists x)\varepsilon X(\mathrm{x}) \textit{ seq } \varepsilon(\exists x)X(x).$$

This non-theorem is a characteristic feature of intuitionism. Brouwer admits only the construction proofs of existence, furnished with an effective algorithm for constructing the object. He excludes proofs that are purely existential, without concreteness, obtained through generalisation, denying the existence of the opposite. As in psychoanalysis, it is not sufficient to know theoretically, even from authoritative treatises, that the unconscious cannot not exist; in order to know that truly it exists, it is necessary to experience it for yourself with your own analysis.

On the other hand, this logic is not obscurantist. In fact, the inverse is valid:

$$\vdash \varepsilon(\exists x)X(\mathrm{x})\ seq\ (\exists x)\varepsilon X(x).$$

If you know that something with a certain property exists, then there exists something of which you know that a certain property is satisfied.

Intransitivity

The need for personal analysis is justified by another intuitionistic non-theorem: unconscious knowledge is intransitive:

$$\dashv \varepsilon(X\ seq\ Y)\ seq\ (\varepsilon X\ seq\ \varepsilon Y).$$

From the fact of knowing in theory that a certain implication is valid, for example, that Y follows from X, it does not automatically ensue that knowledge of X follows from knowledge of Y. Knowledge of the consequence is constructed each time *ex novo*; it is not sufficient to know the antecedent. For analysis it is not sufficient to know *a priori*, because the analysis itself is *a posteriori* knowledge: it is the knowledge of the transition from unconscious to conscious, which is not automatic. In other words, there are two forms of knowledge that do not communicate: theory and practice. One alone is not sufficient. Both are necessary.

Intransitivity implies that unconscious knowledge, even when reasonable, remains strictly subjective[1] [3]. The unconscious does not follow the fashion of intersubjectivity.

Knowledge is not only knowing

The difference between cognitive logic and epistemic logic become radicalised, and at the same time is best expressed, in two theorems that represent the paradigmatic nucleus of two approaches. I consider as emblematic of the cognitive approach Lenzen's *G* system (*G* from *glauben*, to believe) [4]. This is constructed by adding three axioms and a rule of deduction to the classic propositional calculation. Among the axioms is the following:

$$\vdash_{G} Gp\ seq\ non\ G\ non\ p.$$

This is a strictly binary axiom that counters believing with not believing. In fact, it states the *if you believe that p, then you cannot believe that not p*.

It is easy to verify that in intuitionist epistemic logic such a theorem is not valid. Indeed, the converse is valid:

$$\vdash non\ \varepsilon\ non\ p\ seq\ \varepsilon p.$$

In a certain sense this is a fundamental theorem. It states the existence of a nucleus of ignorance within all knowledge; therefore, in particular, there is no knowledge that does not know ~~of~~ something of the negation. In other words, if not reduced to consciousness, that is, to the adjustment of the intellect to the thing, knowledge is supported by an ethical decision: it cuts from the epistemic body something that regards the negation, which turns out thus unknowable in a complete way. Freud's essay 'Negation' (1925), in which negation does not always negate but rather facilitates the return of what had been repressed, enters into this logic.

[1] The intransitivity of unconscious knowledge recalls the intransitivity of the dominance in games with more than two players. Note that our system does not even satisfy the form of epistemic self-transitivity given by the axiom of Gödel-Löb: $\dashv \varepsilon(X\ seq\ X)\ seq\ \varepsilon X$. By defining as *episteme* the formulae *X* for which $\varepsilon X\ seq\ X$, the following interesting negative characterisation of the unconscious is obtained: from the fact that you know the epistemes, it does not follow that you know of them. In this sense, the analyst who works with unconscious knowledge works with a sort of modern and not theological *docta ignorantia*. Our epistemes, understood as units of knowledge, correspond to the notion of signifier without a significance, taken up by Jacques Lacan from the great linguist Ferdinand de Saussure of Geneva.

The double negation

What can be said about the transformation into an epistemic operator of another classic non-intuitionist thesis, for example, the double negation?

As a consequence of the suspension of the axiom of the excluded third, intuitionism loses the law of strong double negation, which permits the cancellation of the double negation:

$$—| \; non\ non\ X\ seq\ X,$$

while maintaining the weak law that permits the introduction of the double negative:

$$|— \; X\ seq\ non\ non\ X.$$

I call δ the operator that transforms each sentence *X* in *not not X seq X*.

δ as an epistemic operator

What properties does δ enjoy? Let's list its theorems and non-theorems.

In general it can be said that δ is an epistemic operator. There are two reasons for this: first of all, it satisfies many theorems in *ε*; in the second place, the epistemic operator *ε* is in a certain sense implicit in δ. In fact, two theorems are valid:

$$|— \; \varepsilon X\ seq\ \delta X,$$

that is, every time that *εX* is true, δ*X* is true, and

$$|— \; \delta X\ aeq\ \varepsilon\delta X.$$

This last theorem establishes the "knowledge" of δ: δ is valid if and only if it is known that δ is valid. Here for the second time we encounter the "uselessness" of knowledge. The discussion becomes simpler, if we drop the requisite of knowledge.

As I said, many theorems that are valid for *ε* are also valid for δ.

Freud's theorem

Kolmogorov's lemma becomes Freud's theorem:

$$|— \; non\ non\ \delta X.$$

This theorem becomes psychoanalytically transparent by interpreting δ as an operator of desire. Thus Freud's thesis states the necessity of desire: we cannot not desire. This interpretation is not contradicted by the theorems that follow.

Oedipus's theorem

The theorem of Socrates becomes that of Oedipus:

$$|— \; \delta\ non\ \delta X$$

Forbidding incest, the Oedipus complex establishes unconscious desire as the desire that should not be desired. But, in effect, it is desired, as the following theorem states.

Lacan's theorem

Descartes's theorem becomes Lacan's theorem, with which it is established that even not desiring is desiring:

$$|— \; non\ \delta X\ seq\ \delta X.$$

All of these theorems regarding negation show that, in intuitionism as in psychoanalysis, negation is weak. It does not always negate, indeed, it sometimes affirms. This phenomenon was declared to be specific to the psychic apparatus by Freud in the essay on *Negation* already mentioned.

Idempotence is not valid

In contrast to the ε operator, the δ operator is not idempotent. The theorem of extension is valid:

$$\vdash \delta X \; seq \; \delta\delta X,$$

but the theorem of absorption is not:

$$\dashv \delta\delta X \; seq \; \delta X.$$

As already mentions, Freud began to construct a psychic apparatus around the first topic, based on the tripartite of conscious, preconscious and unconscious. In the 1920s he proposed another construction, based on the second topic, constituted of the tripartite of Ego, Id, and Super-Ego, where in the Id would be at work a death instinct that leads to the eternal repetition of the identical.

Just as the law of the idempotence of the ε operator causes the collapse of the first topic, so the non-idempotence of the δ operator causes the collapse the second. In fact, the succession of δ operators always produces new δ operators, to infinity: δ^n is different from δ^{n+1}, and the repetition of the identical is never encountered. I consider this to be an improvement of the Freudian metapsychology, in as much as it introduces the discussion of infinity into psychoanalysis.

More about the double negation

In this logic there exists a third operator π (from psyché), based on the double negation. It transforms each sentence X into a weak form of excluded third of the type *nonX vel non nonX* (Jankov's law). π is an epistemic operator in the sense that it is implicit in the epistemic operator ε. In fact, many theorems about the operator δ are formally identical to that of the operator π (double negation, negation that affirms, expansion, etc.), but δ is not implicit in π, nor is π implicit in δ. Therefore, it is necessary to admit the psychic phenomenon that Freud calls *Ichspaltung* (splitting of the ego). In the unconscious there exist two separate and distinct threads of knowledge, independent from one another. In Lacanian terms we could say that one converges in the subject's desire, the other in the desire of the other. The two corresponding epistemic operators represent two different ways in which the unconscious "reacts" to the infinite object.

The subject is finite

I haven't given any proofs of the theorems listed, in that we are dealing with absolutely elementary deductions that can be arrived at in various ways: either with "rule + axiom" formalism in the manner of Frege, or with the "rules only" formalism of Gentzen, Beth and Kleene.

I will use the remaining space-time to give an elementary proof of the finiteness of the subject of science, which is active in the Freudian unconscious.

The subject is finite. This can be proven in various ways.

The ontological proof: The subject's existence ends with death.

The aesthetic proof: Perception of the subject is limited by the perceived object itself, of which the subject perceives always and only one part.

The linguistic proof: The subject of the declaration is finite because every declarative act puts at stake in the "here and now" only a finite number of signifiers.

The logical proof: In my opinion this is the most convincing proof, as long as the logic is epistemic. This is more correct that the aesthetic in that it does not confuse finiteness with limitation. Spinoza had already proven that there exist infinite sets in which the elements are limited, such as the set of distances between circumferences of non-concentric circles.

The proof begins with Descartes. Cartesian doubt, stripped of the rhetorical frills that Descartes liked to dress it up with, reduces to the epistemic alternative: *either I know or I don't know*. The reasoning then continues, "If either I know or I don't know, then I am a subject who doubts." But "either I know or I don't know" is true, thus by *modus ponens* "I am a subject that doubts" is true. We ask ourselves: "When is 'either I know or I don't know" true?" Here we know to answer. "Either I know or I don't know" is an instance of the excluded third in epistemic form. Brouwer has by now incontestably proven that the excluded third *A vel non A* is valid only in finite universe. *Ergo* it is not wrong to state that the subject, who depends on an epistemic form of the excluded third, is finite. The Brouwerian example is simple. If I have two sets A and B and I verify that their union $A \cup B$ is formed of eleven elements, then I can state that affirm that either A is greater than B or B is greater than A, to the exclusion of any third possibilities. That certainty would be diminished if the union had a number of elements that was even or infinite.

The object is infinite

Since the time immemorial of Greek logocentrism, logic appears to be without an object. Mathematics, on the other hand, is not without an object: infinity is the object of mathematics. Because it is more mathematics than logic, intuitionism too works on the basis of the infinite object. This can be confirmed in its semantics that, as Gödel [5] predicted and Kripke [6] realised, avails itself of infinite ordinal models.[2]

From our point of view, it is sufficient to recognise that the relative certainty of the finiteness of the subject is the point of departure for reasonably verifying that the object to which the subject of the science relates, and with it, the subject of the unconscious, is infinite. It presents itself as the spatiotemporally infinite in physics, as biodiversity in biology, as the object of desire in psychoanalysis. But how can we conceive an infinite object? Freud tried to do it: by means of the infinite repetition of the identical. But that is a poor solution. Are there others? Yes, there are infinitely many others. The problem of the infinite is largely indeterminate. Let me explain better.

The infinite is a structure of modern episteme, which Oswald Veblen in 1904 proposed calling non-categorical [7]. This means that the structure in itself cannot be represented – Freud would say that it remains within the primal repression – but it is possible to make partial models or representations of it that are not equivalent to each other. In the (uncertain) terminology of Freud, the models of the structure would be examples of a return from the repression. The eternal repetition of the identical itself is a model of the infinite. This is a different model from numerable infinity, made up of infinite numbers that are all different, and also different from continuous infinity, made up of points so densely stippled that there are no gaps. The infinite repetition of the identical served Freud to explain the existence of an unconscious feeling of guilt, somewhat like a numerable infinity serves to count, and continuous infinity to draw and measure things on earth.

The result of non-categoricalness is interesting for several reasons. First of all, it distinguishes scientific infinity from religious infinity. In fact, religious infinity is single, as testify the great monotheistic religions. On the other hand, scientific infinity is plural, and its plurality conditions two aspects of scientific discourse: indeterminism and self-revision. Scientific

[2] Without going into details about kripkean semantics, I would like to note that an intuitionist model is a countable set of epistemic states, partially ordered by a reflexive and transitive relationship. I also note that for the semantics of classical logic models with a single epistemic state are sufficient.

indeterminism is testified to, for example, by quantum mechanics and by the function of chaos in biology. The acceptance of the indefinite revision of scientific theories is a fact in modern epistemology, from the historic epistemology of Bachelard to the falsificationist epistemology of Popper. Scientific theories are not incontrovertibly and definitively codified in some treatise, but live in the perpetual renovation of the the social bond within the scientific collective. Indeterminism is valid in psychoanalysis as well, for example, in sexual relations;[3] in the same way, it would also be found in the self-corrective recovery of metapsychology in the psychoanalytic discourse, if it were truly scientific and if it lived in the collective of scientific thought. This is what I am hoping for, and I am working so that this happens in these times when it seems to be particularly ‘in’ to talk about the death of psychoanalysis.

The proposal of intuitionism as the mathematics of psychoanalysis exactly suits this purpose.

Bibliography

[1] M. Heidegger (1950) “*The Age of the World Picture*”, in: *The Question Concerning Technology and other Essays*, William Lovitt Ed., Harper Torchbooks, San Francisco 1977, pp. 117-118.

[2] P. Odifreddi (2003) *Il diavolo in cattedra. Logic from Aristotle to Gödel*, Einaudi, Turin, p. 21.

[3] J. von Neumann, O. Morgenstern (1947) *Theory of Games and Economic Behavior*, 2nd ed., Princeton University Press, Princeton, pp. 38-39.

[4] W. Lenzen (1980) *Glauben, Wissen, und Wahrscheinlichkeit. Systeme der epistemischen Logik*, Springer Verlag, Wien, p. 142.

[5] K. Gödel (1986) *On the Intuitionistic Propositional Calculus* (1932), in: i~~n~~ *Kurt Gödel: Collected Works. Volume 1*, edited by S. Feferman, J. W. Dawson, S. C. Kleene, G. H. Moore, R. M. Solovay, and J. van Heijenoort, Oxford University Press, Oxford, 1986, pp. 223-225.

[6] S. Kripke (1965) Semantical analysis of intuitionistic logic, I, in: *Formal systems and recursive functions*, North Holland, Amsterdam, pp. 92-130.

[7] O. Veblen (1904) A System of Axioms for Geometry, *Transactions of the American Mathematical Society*, 5, pp. 343-384.

[3] The sexual relation is indeterminate, that is, it admits infinite solutions, like a system of two equations in two unknowns that differ by a multiplicative constant. There are reasons that justify this as an opportune correction of Jacques Lacan, who rather states the non-existence (or impossibility) of sexual relations, but a discussion of this is beyond the scope of this present paper.